Part II Paper 07: Mathematical Logic 2024-25

Syllabus

- First and second order logic: completeness, compactness, conservativeness, expressive power, and Löwenheim-Skolem theorems, cut-elimination.
- First and second order theories: categoricity, non-standard models of arithmetic.
- Set theory: embedding mathematics in set theory, the cumulative iterative hierarchy, elements of cardinal and ordinal arithmetic, the axiom of choice.
- Recursive functions and computability: decidability, axiomatizability, Church's thesis, Gödel's incompleteness theorems, Hilbert's programme.

Course Outline

This course aims to put the student in a position to assess the philosophical significance of some major results in mathematical logic. Alert attendance at lectures should be considered essential. But the emphasis is not only on the rigorous proof of results but also on philosophical reflection about them.

The first part of the course tackles the ideas of a formal logic and a formal theory with particular attention to the similarities and differences between first-order and second-order logic and arithmetic.

The next part studies set theory, its axioms and motivating conceptions, and the idea that all of mathematics can be embedded in set theory.

In the third part of the course on recursive functions the informal notion of a computable function is described and the relationship, embodied in Church's thesis, between this informal notion and the precise notion of a recursive function is examined. All this leads up to the understanding of Gödel's incompleteness theorems for arithmetic.

Assumed Knowledge

Further Maths A-level or the equivalent is recommended. The material in Part IB Paper 1 Knowledge, Language and World and Part IA Paper 5 Formal Methods is assumed. Knowledge of the material covered by Part IB Paper 2 History of Analytic Philosophy is also desirable.

Objectives

Students will be expected to:

1. Study issues in mathematical logic at an advanced level.
2. Acquire a sophisticated understanding of the scope, purpose and natures of logic.
3. Refine their power of philosophical analysis and argument through study of these ideas.

Reading List

Preliminary Reading

Basic logic

Mathematical Logic
Author: Chiswell, Ian and Wilfrid Hodges; **Type:** Book; **Publication Date:** 2007; **Publisher:** Oxford University Press; **Place of publication:** Oxford; **Tags:** Available online, Recommended reading;

A Friendly Introduction to Mathematical Logic
Author: Leary, Christopher C.; **Type:** Book; **Publication Date:** 2000; **Publisher:** Prentice Hall; **Place of publication:** London; **Tags:** Recommended reading; **Public note:** Chapters 1 - 3. 2nd edition (Geneseo, NY: Milne Library, SUNY Geneseo, 2015) also available;

Set theory

Open Set Theory
Author: Button, Tim; **Type:** Website; **Publication Date:** 2019; **Tags:** Available online, Recommended reading; **Public note:** Available under the "OERs" section;

The Joy of Sets: Fundamentals of Contemporary Set Theory
Author: Devlin, Keith J.; **Type:** Book; **Edition:** 2nd ed.; **Publication Date:** 1993; **Publisher:** Springer-Verlag; **Place of publication:** New York; **Tags:** Available online, Recommended reading; **Public note:** Chapters 1 and 2;

Classic Set Theory: For Guided Independent Study
Author: Goldrei, Derek; **Type:** Book; **Publication Date:** 1996; **Publisher:** Chapman & Hall; **Place of publication:** London; **Tags:** Available online, Recommended reading;

Naïve Set Theory
Author: Halmos, Paul R.; **Type:** Book; **Publication Date:** 1974; **Publisher:** Springer-Verlag; **Place of publication:** New York; **Tags:** Available online, Recommended reading; **Public note:** E-book published in 2017;

Arithmetic, computability etc.

Computability: Computable Functions, Logic, and the Foundations of Mathematics
Author: Epstein, Richard L. and Walter A. Carnielli; **Type:** Book; **Edition:** 3rd ed.; **Publication Date:** 2008; **Publisher:** Advanced Reasoning Forum; **Place of publication:** Socorro; **Tags:** Available online, Recommended reading; **Public note:** The "optional" chapters may be skipped;

Philosophy of Mathematics: Selected Readings
Additional Person Name: edited by Benacerraf, Paul and Hilary Putnam; **Type:** Book; **Edition:** 2nd ed.; **Publication Date:** 1998; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Recommended reading; **Public note:** Classic essays on some of the conceptual issues discussed in the course;

Background: General formal surveys
Two very useful, more discursive surveys, standing back a bit from the nitty gritty of proofs, but trying to give a sense of how results fit together with an indication of their wider significance, are the following. These books will make very useful companions to formal work over the year, and could be especially helpful whenever you feel in danger of not seeing the wood for the trees.

Mathematical Logic and Formalized Theories: A Survey of Basic Concepts and Results
Author: Rogers, Robert; **Type:** Book; **Publication Date:** 1971; **Publisher:** North-Holland; **Place of publication:** Amsterdam; **Tags:** Available online, Recommended reading; **Public note:** Link is to the 2nd edition of the ebook. Rogers's now rather old book is very useful and very accessible though relatively introductory;

A Tour Through Mathematical Logic
Author: Wolf, Robert S.; **Type:** Book; **Publication Date:** 2005; **Publisher:** Mathematical Association of America; **Place of publication:** Washington, DC; **Tags:** Available online, Recommended reading; **Public note:** Wolf's newer book goes further but is a rather bumpier ride because it's somewhat uneven in level of difficulty (though he gives some useful proof sketches);

First Order Logic

Formal expositions

You need to understand the following theorems: The Soundness and Completeness Theorem The Compactness Theorem The Löwenheim-Skolem Theorems You also need to understand how to prove them.

Elementary Predicate Logic

Type: Book Chapter; **Chapter Author:** Hodges, Wilfrid; **Book Title:** Handbook of Philosophical Logic. Volume 1, Elements of Classical Logic; **Pages:** 1 - 131; **Editor:** Gabbay, D. and F. Guenthner; **Publication Date:** 1983; **Publisher:** Reidel; **Place of publication:** Dordrecht; **Tags:** Available online, Recommended reading, Scan available; **Public note:** An expanded version of this chapter appears in the 2nd edition of the Handbook (Dordrecht: Kluwer Academic, 2001). The books by Chiswell/Hodges and by Leary already mentioned of course cover first-order logic in an accessible way. And almost any standard middle or advanced level text will cover the needed ground. But Hodges is a stand-out presentation.;

Mathematical Logic and Formalized Theories: A Survey of Basic Concepts and Results

Author: Rogers, Robert; **Type:** Book; **Publication Date:** 1971; **Publisher:** North-Holland; **Place of publication:** Amsterdam; **Tags:** Available online, Recommended reading; **Public note:** Chapters 2 and 3. A more discursive overview to some main ideas. Link is to the 2nd ed. of the ebook;

Computability and Logic

Author: Boolos, George S., et al.; **Type:** Book; **Edition:** 4th ed.; **Publication Date:** 2002; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Recommended reading; **Public note:** Chapters 9 - 10 and 12 - 14. Another overview treatment highlighting the main ideas, though in more detail. The 5th ed. (2007) available, also online.;

First-Order Logic

Author: Enderton, Herbert B.; **Type:** Book Chapter; **Book Title:** A Mathematical Introduction to Logic; **Book Author:** Enderton, Herbert B.; **Chapter Number:** 2; **Pages:** 67 - 181; **Edition:** 2nd ed.; **Publication Date:** 2001; **Publisher:** Harcourt; **Place of publication:** San Diego; **Tags:** Available online, Further reading; **Public note:** Another standard textbook treatment, for enthusiasts.;

Introduction to Mathematical Logic

Author: Mendelson, Elliott; **Type:** Book; **Edition:** 4th ed.; **Publication Date:** 1997; **Publisher:** Chapman & Hall; **Place of publication:** London; **Tags:** Further reading; **Public note:** Sections 2.1 - 2.9. Another standard textbook treatment, for enthusiasts;

Philosophical issues arising: Skolem's paradox

Skolem's Paradox

Author: Bays, Timothy; **Additional Person Name:** edited by Zalta, Edward N.; **Type:** Website; **Publisher:** Stanford Encyclopedia of Philosophy (Winter2014 edition); **Tags:** Available online, Recommended reading; **Public note:** Basic discussion;

Blitz on Paradise

Author: Giaquinto, Marcus; **Type:** Book Chapter; **Book Title:** The Search for Certainty: A Philosophical Account of Foundations of Mathematics; **Book Author:** Giaquinto, Marcus; **Chapter Number:** 4, sect.2; **Pages:** 130 - 136; **Publication Date:** 2002; **Publisher:** Clarendon Press; **Place of publication:** Oxford; **Tags:** Available online, Recommended reading; **Public note:** Basic discussion;

Some Remarks on Axiomatised Set Theory (especially section 3)

Type: Book Chapter; **Chapter Author:** Skolem, Thoralf; **Book Title:** From Frege to Gödel: A Source Book in Mathematical Logic, 1879-1931; **Pages:** 290 - 301; **Editor:** Van Heijenoort, Jean; **Publication Date:** 1967; **Publisher:** Harvard University Press; **Place of publication:** Cambridge; **Tags:** Available online, Further reading, Scan available; **Public note:** Classic paper, although hard;

Models and Reality
The Journal of Symbolic Logic
Author: Putnam, Hilary; **Type:** Article; **Pages:** 464 - 482; **Publication Date:** 1980; **Volume:** 45; **Issue:** 3; **Tags:** Available online, Further reading; **Public note:** Classic paper, although hard. Do not be alarmed if the Epistemological/Logical Digression makes no sense to you!;

Philosophical issues arising: Skolem's paradox: further reading

Skolem's paper is the locus classicus of the paradox; Putnam rejuvenated philosophical discussion of the paradox. To start understanding them, read the remainder of Bays's 'Skolem's Paradox,' i.e. sections 3-4. Then look into the following.

Skolem and the Skeptic
Proceedings of the Aristotelian Society Supplementary Volumes
Author: Benacerraf, Paul and Crispin Wright; **Type:** Article; **Pages:** 117 - 137; **Publication Date:** 1985; **Volume:** 59; **Issue:** 1; **Tags:** Available online, Further reading;

Philosophy and Model Theory
Author: Button, Tim and Sean Walsh; **Type:** Book; **Publication Date:** 2018; **Publisher:** Oxford University Press; **Place of publication:** Oxford; **Tags:** Available online, Further reading; **Public note:** Chapters 6 - 8, focussing on chapter 7;

Skolem and the Löwenheim-Skolem Theorem: A Case Study of the Philosophical Significance of Mathematical Results
History and Philosophy of Logic
Author: George, Alexander; **Type:** Article; **Pages:** 75 - 89; **Publication Date:** 1985; **Volume:** 6; **Issue:** 1; **Tags:** Available online, Further reading;

Intended Models and the Löwenheim-Skolem Theorem
Journal of Philosophical Logic
Author: Klenk, Virginia; **Type:** Article; **Pages:** 475 - 489; **Publication Date:** 1976; **Volume:** 5; **Issue:** 4; **Tags:** Available online, Further reading;

Skolem's Criticisms of Set Theory
Noûs
Author: McIntosh, Clifton; **Type:** Article; **Pages:** 313 - 334; **Publication Date:** 1979; **Volume:** 13; **Issue:** 3; **Tags:** Available online, Further reading;

The Significance of Non-Standard Models
Analysis
Author: Melia, Joseph; **Type:** Article; **Pages:** 127 - 134; **Publication Date:** 1995; **Volume:** 55; **Issue:** 3; **Tags:** Available online, Further reading;

The Löwenheim-Skolem Theorem
Author: Moore, Adrian W.; **Type:** Book Chapter; **Book Title:** The Infinite; **Book Author:** Moore, Adrian W.; **Chapter Number:** 11; **Pages:** 159 - 171; **Publication Date:** 1990; **Publisher:** Routledge; **Place of publication:** London; **Tags:** Available online, Further reading;

Second Order Logic and Rule Following
Author: Shapiro, Stewart; **Type:** Book Chapter; **Book Title:** Foundations Without Foundationalism: A Case for Second-Order Logic; **Book Author:** Shapiro, Stewart; **Chapter Number:** 8; **Pages:** 203 - 219; **Publication Date:** 1991; **Publisher:** Clarendon Press; **Place of publication:** Oxford; **Tags:** Available online, Further reading; **Public note:** For further discussion;

In Defense of Putnam's Brains
Philosophical Studies
Author: Tymoczko, Thomas; **Type:** Article; **Pages:** 281 - 297; **Publication Date:** 1989; **Volume:** 57; **Issue:** 3; **Tags:** Available online, Further reading; **Public note:** For further discussion;

Skolem and the Skeptic
Proceedings of the Aristotelian Society Supplementary Volumes
Author: Benacerraf, Paul and Crispin Wright; **Type:** Article; **Pages:** 85 - 137; **Publication Date:** 1985; **Volume:** 59; **Issue:** 1; **Tags:** Available online, Further reading; **Public note:** For further discussion;

Technical and philosophical issues arising: Cut elimination

Cut-elimination results are typically proved for proof systems presented in sequent calculus form, rather than the natural deduction forms you have studied. It is therefore advised to gain some familiarity with sequent calculus presentations first. For a good presentation of such systems, their relation to natural deduction, and cut-elimination, see:

The Philosophical Basis of Intuitionistic Logic
Type: Book Chapter; **Chapter Author:** Dummett, Michael; **Book Title:** Truth and Other Enigmas; **Pages:** 290 - 318; **Publication Date:** 1978; **Publisher:** Duckworth; **Place of publication:** London; **Tags:** Available online, Recommended reading; **Public note:** Link is to reprint in Philosophy of Mathematics: selected readings edited by Paul Benacerraf and Hilary Putnam, pp. 97-129;

Structural Proof Theory
Author: Negri, Sara and Jan von Plato; **Type:** Book; **Publication Date:** 2001; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Recommended reading; **Public note:** Chapters 1 and 2;

Don't Eliminate Cut
Journal of Philosophical Logic
Author: Boolos, George; **Type:** Article; **Pages:** 373 - 378; **Publication Date:** 1984; **Volume:** 13; **Issue:** 4; **Tags:** Available online, Recommended reading; **Public note:** The classic on the formal significance of cut-elimination;

What Is Logic?
The Journal of Philosophy
Author: Hacking, Ian; **Type:** Article; **Pages:** 285 - 319; **Publication Date:** 1979; **Volume:** 76; **Issue:** 6; **Tags:** Available online, Recommended reading; **Public note:** The philosophical significance of cut-elimination, along with the closely related ideas of normalization and harmony;

An Introduction to Proof Theory: Normalization, Cut-Elimination, and Consistency Proofs
Author: Mancosu, Paolo, et al.; **Type:** Book; **Publication Date:** 2021; **Publisher:** Oxford University Press; **Place of publication:** Oxford; **Tags:** Available online, Recommended reading;

Second Order Logic

Formal expositions

You need some sense of the difference between first-order logic (full) and second-order logic in terms of axiomatizability, compactness, the Löwenheim-Skolem theorems, etc. You'll need to understand why e.g. second-order Peano arithmetic is categorical (with the full semantics) and first-order Peano arithmetic isn't.

Second Order and Higher-Order Logic
Author: Enderton, Herbert B.; **Additional Person Name:** edited by Zalta, Edward N.; **Type:** Website; **Publisher:** Stanford Encyclopedia of Philosophy (Fall 2015 edition); **Tags:** Available online, Recommended reading; **Public note:** Useful introductory overview;

Foundations Without Foundationalism: A Case for Second-Order Logic
Author: Shapiro, Stewart; **Type:** Book; **Publication Date:** 1991; **Publisher:** Clarendon Press; **Place of publication:** Oxford; **Tags:** Available online, Recommended reading; **Public note:** Chapters 3 - 5. The classic modern presentation. This will give you more than enough. That said, you might find Shapiro's approach to formal matters a little unfamiliar. So, for alternative treatments - including the contrast between first- and second-order logic - dip into any of the following works.;

Second-Order Logic
Author: Boolos, George, John P. Burgess and Richard C. Jeffrey; **Type:** Book Chapter; **Book Title:** Computability and Logic; **Book Author:** Boolos, George, John P. Burgess and Richard C. Jeffrey; **Chapter Number:** 22; **Pages:** 279 - 285; **Edition:** 4th ed.; **Publication Date:** 2002; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Further reading; **Public note:** The 5th ed. (2007) available, also online;

Philosophy and Model Theory
Author: Button, Tim and Sean Walsh; **Type:** Book; **Publication Date:** 2018; **Publisher:** Oxford University Press; **Place of publication:** Oxford; **Tags:** Available online, Further reading; **Public note:** Chapter 1, and sections 4.1, 7.3 - 7.6;

Second Order Logic
Author: Van Dalen, Dirk; **Type:** Book Chapter; **Book Title:** Logic and Structure; **Book Author:** Van Dalen, Dirk; **Pages:** 145 - 153; **Edition:** 5th ed.; **Publication Date:** 2013; **Publisher:** Springer; **Place of publication:** London; **Tags:** Available online, Further reading;

The Diagonalization Lemma
Author: Smith, Peter; **Type:** Book Chapter; **Book Title:** An Introduction to Gödel's Theorems; **Book Author:** Smith, Peter; **Chapter Number:** 21 (2007 ed.) or 24 (2013 ed.); **Publication Date:** 2007; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Further reading; **Public note:** More specific discussion on first- vs. second-order arithmetic. 2nd edition (2013) also available (in print and online). Link is to the 2013 ed. of the ebook.;

Philosophical issues arising 1. On the status of second-order logic as logic

The Scope of Logic
Author: Quine, Willard V.O.; **Type:** Book Chapter; **Book Title:** Philosophy of Logic; **Book Author:** Quine, Willard V.O.; **Chapter Number:** 5; **Pages:** 61 - 79; **Edition:** 2nd ed.; **Publication Date:** 1986; **Publisher:** Harvard University Press; **Place of publication:** Cambridge; **Tags:** Available online, Recommended reading; **Public note:** Is second-order logic just set-theory in disguise (with the second-order quantifiers running over sets)? That's the view of Quine.;

On Second-Order Logic
The Journal of Philosophy
Author: Boolos, George S.; **Type:** Article; **Pages:** 509 - 527; **Publication Date:** 1975; **Volume:** 72; **Issue:** 16; **Tags:** Available online, Recommended reading; and in S. Shapiro, ed., The Limits of Logic (Aldershot: Dartmouth, 1996).; **Public note:** For discussion;

Foundations without Foundationalism: A Case for Second-Order Logic
Author: Shapiro, Stewart; **Type:** Book; **Publication Date:** 1991; **Publisher:** Clarendon Press; **Place of publication:** Oxford; **Tags:** Available online, Recommended reading; **Public note:** Read chapter 2, sections 3-5. For discussion;

A Curious Inference
Journal of Philosophical Logic
Author: Boolos, George S.; **Type:** Article; **Pages:** 1 - 12; **Publication Date:** 1987; **Volume:** 16; **Issue:** 1; **Tags:** Available online, Further reading; and in S. Shapiro, ed., The Limits of Logic (Aldershot: Dartmouth, 1996); **Public note:** Also in his Logic, Logic and Logic (Cambridge, MA: Harvard University Press, 1998); and in S. Shapiro, ed., The Limits of Logic (Aldershot: Dartmouth, 1996);

Which Logic Is the Right Logic?
Synthese
Author: Tharp, Leslie H.; **Type:** Article; **Pages:** 1 - 21; **Publication Date:** 1975; **Volume:** 31; **Issue:** 1; **Tags:** Available online, Further reading; **Public note:** Also in S. Shapiro, ed., The Limits of Logic (Aldershot: Dartmouth, 1996);

Neutralism within the Semantic Tradition
Thought: A Journal of Philosophy
Author: Trueman, Robert; **Type:** Article; **Pages:** 246 - 251; **Publication Date:** 2012; **Volume:** 1; **Issue:** 3; **Tags:** Available online, Further reading;

Second-Order Logic and Foundations of Mathematics
The Bulletin of Symbolic Logic
Author: Väänänen, Jouko; **Type:** Article; **Pages:** 504 - 520; **Publication Date:** 2001; **Volume:** 7; **Issue:** 4; **Tags:** Available online, Further reading;

Philosophical issues arising 2. The connections with plural quantification and natural language

George Boolos has argued that we can “tame” second-order logic (and see it as genuinely part of logic) by interpreting second-order quantifiers as (akin to) plural quantifiers. For a basic exchange, see Boolos and Resnik:

To Be Is to Be a Value of a Variable (or to Be Some Values of Some Variables)
The Journal of Philosophy
Author: Boolos, George S.; **Type:** Article; **Pages:** 430 - 449; **Publication Date:** 1984; **Volume:** 81; **Issue:** 8; **Tags:** Available online, Recommended reading; and in S. Shapiro, ed., The Limits of Logic (Aldershot: Dartmouth, 1996); **Public note:** Reprinted in his Logic, Logic and Logic (Cambridge, MA: Harvard University Press, 1998); and in S. Shapiro, ed., The Limits of Logic (Aldershot: Dartmouth, 1996);

Second-Order Logic Still Wild
The Journal of Philosophy
Author: Resnik, Michael D.; **Type:** Article; **Pages:** 75 - 87; **Publication Date:** 1988; **Volume:** 85; **Issue:** 2; **Tags:** Available online, Recommended reading; **Public note:** Reprinted in S. Shapiro, ed., The Limits of Logic (Aldershot: Dartmouth, 1996);

Nominalist Platonism
The Philosophical Review
Author: Boolos, George S.; **Type:** Article; **Pages:** 327 - 344; **Publication Date:** 1985; **Volume:** 94; **Issue:** 3; **Tags:** Available online, Further reading; and in S. Shapiro, ed., The Limits of Logic (Aldershot: Dartmouth, 1996); **Public note:** Reprinted in his Logic, Logic and Logic (Cambridge, MA: Harvard University Press, 1998); and in S. Shapiro, ed., The Limits of Logic (Aldershot: Dartmouth, 1996);

On Higher-Order Logic and Natural Language
Type: Book Chapter; **Chapter Author:** Higginbotham, James; **Book Title:** Philosophical Logic; **Pages:** 1 - 27; **Editor:** Smiley, Timothy J.; **Publication Date:** 1998; **Publisher:** Oxford University Press; **Place of publication:** Oxford; **Tags:** Available online, Further reading; **Public note:** Link is to the original publication in The Proceedings of the British Academy, 95.;

Plural Quantification
Author: Linnebo, Øystein; **Additional Person Name:** edited by Zalta, Edward, N.; **Type:** Website; **Publisher:** Stanford Encyclopedia of Philosophy (Summer 2017 edition); **Tags:** Available online, Further reading;

Nominalism Through De-Nominalization
Noûs
Author: Rayo, Agustin and Stephen Yablo; **Type:** Article; **Pages:** 74 - 92; **Publication Date:** 2001; **Volume:** 35; **Issue:** 1; **Tags:** Available online, Further reading;

Set Theory

Start with the book by Tim Button (below), which covers all the technicalities. It also gives a quick introduction to several of the philosophical issues.

Open Set Theory
Author: Button, Tim; **Type:** Website; **Publication Date:** 2019; **Tags:** Available online, Recommended reading; **Public note:** Available under the "OERs" section;

Formal expositions

Different people will respond to different formal texts; so if you want alternative treatments, the George and Wolf texts outline some key ideas behind ZFC. Devlin, Goldrei, and Halmos are excellent, fuller treatments of axiomatic set theory.

Set Theory

Author: George, Alexander and Daniel J. Velleman; **Type:** Book Chapter; **Book Title:** Philosophies of Mathematics; **Book Author:** George, Alexander and Daniel J. Velleman; **Chapter Number:** 3; **Pages:** 44 - 88; **Publication Date:** 2002; **Publisher:** Blackwell; **Place of publication:** Oxford; **Tags:** Recommended reading;

Axiomatic Set Theory

Author: Wolf, Robert S.; **Type:** Book Chapter; **Book Title:** A Tour Through Mathematical Logic; **Book Author:** Wolf, Robert S.; **Chapter Number:** 2; **Pages:** 59 - 94; **Publication Date:** 2005; **Publisher:** Mathematical Association of America; **Place of publication:** Washington; **Tags:** Available online, Recommended reading;

The Joy of Sets: Fundamentals of Contemporary Set Theory

Author: Devlin, Keith J.; **Type:** Book; **Edition:** 2nd ed.; **Publication Date:** 1993; **Publisher:** Springer-Verlag; **Place of publication:** New York; **Tags:** Available online, Recommended reading; **Public note:** Chapters 1 - 3. Excellent, fuller treatment of axiomatic set theory;

Classic Set Theory: For Guided Independent Study

Author: Goldrei, Derek; **Type:** Book; **Publication Date:** 1996; **Publisher:** Chapman & Hall; **Place of publication:** London; **Tags:** Available online, Recommended reading; **Public note:** Another excellent, fuller treatment of axiomatic set theory;

Naïve Set Theory

Author: Halmos, Paul R.; **Type:** Book; **Publication Date:** 1974; **Publisher:** Springer-Verlag; **Place of publication:** New York; **Tags:** Available online, Recommended reading; **Public note:** Yet another excellent, fuller treatment of axiomatic set theory. E-book published in 2017;

Discovering Modern Set Theory

Author: Just, Winfried and Martin Weese; **Type:** Book; **Publication Date:** 1996; **Publisher:** American Mathematical Society; **Place of publication:** Providence; **Tags:** Available online, Recommended reading; **Public note:** Another modern text that is written in a relaxed style, and is often extremely helpful in the way it introduces concepts and theorems;

Set Theory and Its Philosophy: A Critical Introduction

Author: Potter, Michael D.; **Type:** Book; **Publication Date:** 2004; **Publisher:** Oxford University Press; **Place of publication:** Oxford; **Tags:** Available online, Recommended reading; **Public note:** A lovely way of thinking about set theory – coming to be known as "the Scott-Potter theory" – is presented in this text (though be warned that this is indeed a somewhat unusual approach to the technicalities);

Historical background

If you want to know about the history of ZFC, try looking at the three references listed below.

The Early Development of Set Theory

Author: Ferreirós, José; **Additional Person Name:** edited by Zalta, Edward N.; **Type:** Website; **Publisher:** Stanford Encyclopedia of Philosophy (Summer 2019 edition); **Tags:** Available online, Recommended reading;

Foundations of Set Theory

Author: Fraenkel, Abraham A. and Yehoshua Bar-Hillel; **Type:** Book; **Publication Date:** 1958; **Publisher:** North-Holland; **Place of publication:** Amsterdam; **Tags:** Available online, Recommended reading; **Public note:** Chapters 1 and 2;

Investigations in the Foundations of Set Theory I

Type: Book Chapter; **Chapter Author:** Zermelo, Ernst; **Book Title:** From Frege to Gödel: a source book in mathematical logic, 1879-1931; **Pages:** 199 - 215; **Editor:** Van Heijenoort, Jean; **Publication Date:** 1967; **Publisher:** Harvard University Press; **Place of publication:** Cambridge; **Tags:** Available online, Recommended reading, Scan available;

The Mathematical Development of Set Theory from Cantor to Cohen
The Bulletin of Symbolic Logic
Author: Kanamori, Akihiro; **Type:** Article; **Pages:** 1 - 71; **Publication Date:** 1996; **Volume:** 2; **Issue:** 1; **Tags:** Further reading; **Public note:** Those who find the historical stories fascinating - and the illuminate why one particular set theory has ended up as the canonical one - can follow up Ferreiros by dipping into at least the first half of the longer story here.;

Philosophical issues arising 1. Set theory as a foundation for mathematics
In what sense can we say that set theory "provides a foundation for" mathematics?

The Search for Certainty: A Philosophical Account of Foundations of Mathematics
Author: Giaquinto, Marcus; **Type:** Book; **Publication Date:** 2002; **Publisher:** Clarendon Press; **Place of publication:** Oxford; **Tags:** Available online, Recommended reading; **Public note:** Part 1 and Part 5, sections 1 and 2. An introductory discussion of what set theory is supposed to do for us;

Realism
Author: Maddy, Penelope; **Type:** Book Chapter; **Book Title:** Naturalism in Mathematics; **Book Author:** Maddy, Penelope; **Chapter Number:** Part 2; **Publication Date:** 1997; **Publisher:** Clarendon Press; **Place of publication:** Oxford; **Tags:** Available online, Further reading; **Public note:** Further discussion.;

What is Required of a Foundation for Mathematics?
Philosophia Mathematica
Author: Mayberry, John; **Type:** Article; **Pages:** 16 - 35; **Publication Date:** 1994; **Volume:** 2; **Issue:** 1; **Tags:** Available online, Further reading; **Public note:** Further discussion.;

What Are Sets and What Are They For?
Philosophical Perspectives, Metaphysics
Author: Oliver, Alex and Timothy Smiley; **Type:** Article; **Pages:** 123 - 155; **Publication Date:** 2006; **Volume:** 20; **Issue:** 1; **Tags:** Available online, Further reading; **Public note:** Some subversive remarks.;

Philosophical issues arising 2. What conception of sets is supposed to be reflected in standard set theories? Does that conception justify the axioms?

The Iterative Conception of Set
The Journal of Philosophy
Author: Boolos, George; **Type:** Article; **Pages:** 215 - 231; **Publication Date:** 1971; **Volume:** 68; **Issue:** 8; **Tags:** Available online, Recommended reading;

Iteration Again
Philosophical Topics
Author: Boolos, George; **Type:** Article; **Pages:** 5 - 21; **Publication Date:** 1989; **Volume:** 17; **Issue:** 2; **Tags:** Available online, Recommended reading;

What Is the Iterative Conception of Set?
Type: Book Chapter; **Chapter Author:** Parsons, Charles; **Book Title:** Philosophy of Mathematics: Selected Readings; **Pages:** 503 - 529; **Editor:** Paul Benacerraf and Hilary Putnam; **Edition:** 2nd ed.; **Publication Date:** 1998; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Recommended reading;

The Iterative Conception of Set
The Review of Symbolic Logic
Author: Forster, Thomas; **Type:** Article; **Pages:** 97 - 110; **Publication Date:** 2008; **Volume:** 1; **Issue:** 1; **Tags:** Available online, Further reading;

What is Cantor's Continuum Problem?
The American Mathematical Monthly
Author: Gödel, Kurt; **Type:** Article; **Pages:** 515 - 525; **Publication Date:** 2018; **Volume:** 54; **Issue:** 9; **Tags:** Available online, Further reading; and in P. Benacerraf & H. Putnam, eds., Philosophy of Mathematics: Selected Readings. 2nd ed. (Cambridge: Cambridge University Press, 1983). Also available online at: https://doi.org/10.1017/CBO9781139171519.025;

Boolos on the Justification of Set Theory
Philosophia Mathematica
Author: Paseau, Alexander; **Type:** Article; **Pages:** 30 - 53; **Publication Date:** 2007; **Volume:** 15; **Issue:** 1; **Tags:** Available online, Further reading;

Iterative Set Theory
The Philosophical Quarterly
Author: Potter, Michael D.; **Type:** Article; **Pages:** 178 - 193; **Publication Date:** 1993; **Volume:** 43; **Issue:** 171; **Tags:** Available online, Further reading;

The Concept of Set
Author: Wang, Hao; **Type:** Book Chapter; **Book Title:** From Mathematics to Philosophy; **Book Author:** Wang, Hao; **Pages:** 181 - 223; **Publication Date:** 1974; **Publisher:** Routledge & Kegan Paul; **Place of publication:** London; **Tags:** Available online, Further reading; **Public note:** Link is to the alternative source (Philosophy of Mathematics, P. Benacerraf and H. Putnam, eds. (Cambridge: CUP, 1983), pp. 530-570);

Gödel's First Incompleteness Theorem

Formal expositions

Gödel's Theorem an Incomplete Guide to Its Use and Abuse
Author: Franzén, Torkel; **Type:** Book; **Publication Date:** 2005; **Publisher:** A.K. Peters; **Place of publication:** Wellesley; **Tags:** Available online, Recommended reading; **Public note:** Chapters 1 - 3. A nice introduction (in a splendidly sane short book, which you should eventually read all of);

Incompleteness. Undecidability
Author: Rogers, Robert; **Type:** Book Chapter; **Book Title:** Mathematical Logic and Formalized Theories: A Survey of Basic Concepts and Results; **Book Author:** Rogers, Robert; **Chapter Number:** 8; **Pages:** 186 - 229; **Publication Date:** 1971; **Publisher:** North-Holland; **Place of publication:** Amsterdam; **Tags:** Available online, Recommended reading; **Public note:** Link is to the 2nd ed. of the ebook. Another introductory survey;

Incompleteness Theorems
Author: George, Alexander and Daniel J. Velleman; **Type:** Book Chapter; **Book Title:** Philosophies of Mathematics; **Book Author:** George, Alexander and Daniel J. Velleman; **Chapter Number:** 7; **Pages:** 173 - 213; **Publication Date:** 2002; **Publisher:** Blackwell; **Place of publication:** Oxford; **Tags:** Available online, Recommended reading, Scan available; **Public note:** A bit more detail;

An Introduction to Gödel's Theorems
Author: Smith, Peter; **Type:** Book; **Edition:** 2nd ed.; **Publication Date:** 2013; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Recommended reading; **Public note:** 1st edition (2007) also available. Especially chapters 16 and 17 (in 2nd ed. chapters 21 and 22), which more or less follow Gödel's original proof. This is a full-dress proof with all the trimmings.;

Computability and Logic
Author: Boolos, George S., et al.; **Type:** Book; **Edition:** 4th ed.; **Publication Date:** 2002; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Further reading; **Public note:** Gets to Godel's Theorem in chapter 17. Godel proved his First Theorem in 1931, before the beginnings of the general theory of computability really got underway in 1936: the original version of the Theorem appeals only to the restricted notion of a 'primitive recursive' function. Many modern books, however, approach things in a non-historical order, first explaining the general theory of computability, and then moving on to Godel's Theorem. Two notable books which do things this way round are this one and Epstein & Carnielli, below. The 5th ed. (2007) available, also online (https://idiscover.lib.cam.ac.uk/permalink/f/5hbpu5/44CAM_ALMA51527602460003606);

Computability: Computable Functions, Logic, and the Foundations of Mathematics
Author: Epstein, Richard L. and Walter A. Carnielli; **Type:** Book; **Edition:** 3rd ed.; **Publication Date:** 2008; **Publisher:** Advanced Reasoning Forum; **Place of publication:** Socorro; **Tags:** Available online, Further reading;

Philosophical issues arising 1. Minds and machines

Minds, Machines and Gödel
Philosophy
Author: Lucas, John R.; **Type:** Article; **Pages:** 112 - 127; **Publication Date:** 1961; **Volume:** 36; **Issue:** 137; **Tags:** Available online, Recommended reading; **Public note:** Lucas famously argues that Gödel's theorem shows that minds are not machines. (It is not really essential, but might help if you know what Turing machine is before you start reading this debate).;

Minds and Machines
Author: Putnam, Hilary; **Type:** Book Chapter; **Book Title:** Philosophical Papers. Volume 2. Mind, Language and Reality; **Book Author:** Putnam, Hilary; **Chapter Number:** 18; **Pages:** 362 - 385; **Publication Date:** 1975; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Recommended reading; **Public note:** A classic riposte to Lucas above;

Shadows of the Mind: A Search for the Missing Science of Consciousness
Author: Penrose, Roger; **Type:** Book; **Publication Date:** 1994; **Publisher:** Oxford University Press; **Place of publication:** Oxford; **Tags:** Recommended reading; **Public note:** Chapters 2 and 3, especially sections 2.5 - 3.10. One of those who have tried to rescue Lucas's argument.;

Penrose's Gödelian Argument
Psyche
Author: Feferman, Soloman; **Type:** Article; **Pages:** 21 - 32; **Publication Date:** 1995; **Volume:** 2; **Issue:** 7; **Tags:** Available online, Recommended reading; **Public note:** There's much more on Penrose to be found in the same issue of Psyche at: http://journalpsyche.org/archive/volume-2-1995-1996/. A stern critique of Penrose (above);

Some Basic Theorems in the Foundations of Mathematics and Their Philosophical Implications
Author: Gödel, Kurt; **Type:** Book Chapter; **Book Title:** Collected Works. Volume 3; **Book Author:** Gödel, Kurt; **Pages:** 304 - 323; **Publication Date:** 1986; **Publisher:** Oxford University Press; **Place of publication:** Oxford; **Tags:** Available online, Further reading; **Public note:** Another related discussion. This, the "Gibbs Lecture" from 1951, is not easy but is remarkably rich;

Lucas against Mechanism
Philosophy
Author: Lewis, David; **Type:** Article; **Pages:** 231 - 233; **Publication Date:** 2009; **Volume:** 44; **Issue:** 169; **Tags:** Available online, Further reading;

The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics
Author: Penrose, Roger; **Type:** Book; **Publication Date:** 1990; **Publisher:** Oxford University Press; **Place of publication:** Oxford; **Tags:** Available online, Further reading; **Public note:** Especially pp. 129-146 and 538-141 - this is Penrose's first shot at extracting philosophical morals from Gödel, in an earlier book;

An Introduction to Gödel's Theorems
Author: Smith, Peter; **Type:** Book; **Edition:** 2nd ed.; **Publication Date:** 2013; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Further reading; **Public note:** 1st edition (2007) also available. Section 28.6 (37.6 in 2nd ed.) - relates to the argument in Gödel's paper which springs from the Second Incompleteness Theorem;

Philosophical issues arising 2. Is the notion of natural number open-ended?

The Philosophical Significance of Gödel's Theorem
Ratio
Author: Dummett, Michael; **Type:** Article; **Pages:** 140 - 155; **Publication Date:** 1963; **Volume:** 5; **Tags:** Available online, Recommended reading, Scan available;

More on "The Philosophical Significance of Gödel's Theorem"
Type: Book Chapter; **Chapter Author:** Moore, Adrian W.; **Book Title:** New Essays on the Philosophy of Michael Dummett; **Pages:** 103 - 126; **Editor:** Brandl, J. L. and Sullivan, P.M.; **Publication Date:** 1998; **Publisher:** Rodopi; **Place of publication:** Amsterdam; **Tags:** Available online, Recommended reading, Scan available;

About "The Philosophical Significance of Gödel's Theorem": Some Issues
Author: Wright, Crispin; **Type:** Book Chapter; **Book Title:** Realism, Meaning and Truth; **Book Author:** Wright, Crispin; **Pages:** 321 - 354; **Publication Date:** 1987; **Publisher:** Basil Blackwell; **Place of publication:** Oxford; **Tags:** Available online, Recommended reading, Scan available;

Gödel's Second Incompleteness Theorem

Formal expositions

Incompleteness Theorems
Author: George, Alexander and Daniel J. Velleman; **Type:** Book Chapter; **Book Title:** Philosophies of Mathematics; **Book Author:** George, Alexander and Daniel J. Velleman; **Chapter Number:** 7; **Pages:** 173 - 213; **Publication Date:** 2002; **Publisher:** Blackwell; **Place of publication:** Oxford; **Tags:** Available online, Recommended reading, Scan available; **Public note:** A useful place to start;

An Introduction to Gödel's Theorems
Author: Smith, Peter; **Type:** Book; **Edition:** 2nd ed.; **Publication Date:** 2013; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Recommended reading; **Public note:** Chapters 31-35 in 2nd ed. (chapters 24-26 in 1st ed.). That should put you in a position to appreciate Boolos's wonderful jeu d'esprit, below.;

Gödel's Second Incompleteness Theorem Explained in Words of One Syllable
Mind
Author: Boolos, George; **Type:** Article; **Pages:** 1 - 3; **Publication Date:** 1994; **Volume:** 103; **Issue:** 409; **Tags:** Available online, Recommended reading;

What Does Gödel's Second Incompleteness Theorem Show?
Noûs
Author: Moore, A. W.; **Type:** Article; **Pages:** 573 - 584; **Publication Date:** 1988; **Volume:** 22; **Issue:** 4; **Tags:** Available online, Recommended reading; **Public note:** Useful commentary;

Hilbert's Programme
As we'll see, the main philosophical issue arising from Gödel's Second Theorem (at least as far as this paper is concerned) is its impact on Hilbert's Programme.

The Search for Certainty: A Philosophical Account of Foundations of Mathematics
Author: Giaquinto, Marcus; **Type:** Book; **Publication Date:** 2002; **Publisher:** Clarendon Press; **Place of publication:** Oxford; **Tags:** Available online, Recommended reading; **Public note:** Part 4, sects. 3 and 4. A very good introduction to Hilbert;

On the Infinite
Type: Book Chapter; **Chapter Author:** Hilbert, David; **Book Title:** Philosophy of Mathematics: Selected Readings; **Pages:** 183 - 201; **Editor:** Benacerraf , Paul and Hilary Putnam; **Edition:** 2nd ed.; **Publication Date:** 1998; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Recommended reading; **Public note:** Hilbert's work - do read the man himself;

Finitism
Author: George, Alexander, and Daniel J. Velleman; **Type:** Book Chapter; **Book Title:** Philosophies of Mathematics; **Book Author:** George, Alexander, and Daniel J. Velleman; **Chapter Number:** 6; **Pages:** 147 - 172; **Publication Date:** 2002; **Publisher:** Blackwell; **Place of publication:** Oxford; **Tags:** Further reading; **Public note:** Further elaboration on Hilbert;

'Hilbert's Programme'
Dialectica
Author: Kreisel, Georg; **Type:** Article; **Pages:** 346 - 372; **Publication Date:** 1958; **Volume:** 12; **Issue:** 3/4; **Tags:** Available online, Further reading;

Finitism and Intuitive Knowledge
Type: Book Chapter; **Chapter Author:** Parsons, Charles; **Book Title:** The Philosophy of Mathematics Today; **Pages:** 249 - 270; **Editor:** Schirn, M.; **Publication Date:** 1998; **Publisher:** Clarendon Press; **Place of publication:** Oxford; **Tags:** Available online, Further reading, Scan available;

Hilbert's Programme
Author: Potter, Michael D.; **Type:** Book Chapter; **Book Title:** Reason's Nearest Kin: Philosophies of Arithmetic from Kant to Carnap; **Book Author:** Potter, Michael D.; **Publication Date:** 2002; **Publisher:** Oxford University Press; **Place of publication:** Oxford; **Tags:** Available online, Further reading;

Finitism
The Journal of Philosophy
Author: Tait, W. W; **Type:** Article; **Pages:** 524 - 546; **Publication Date:** 1981; **Volume:** 78; **Issue:** 9; **Tags:** Available online, Further reading;

Philosophical issue arising: What was Hilbert's Programme? Do Gödel's incompleteness theorems undermine it?

Underivability of "Consistency"
Author: Giaquinto, Marcus; **Type:** Book Chapter; **Book Title:** The Search for Certainty: A Philosophical Account of Foundations of Mathematics; **Book Author:** Giaquinto, Marcus; **Chapter Number:** 5. Section 2; **Pages:** 182 - 198; **Publication Date:** 2002; **Publisher:** Clarendon Press; **Place of publication:** Oxford; **Tags:** Available online, Recommended reading; **Public note:** A standard answer to the second question in the section title is given in this reference;

An Introduction to Gödel's Theorems
Author: Smith, Peter; **Type:** Book; **Edition:** 2nd ed.; **Publication Date:** 2013; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Recommended reading; **Public note:** Another standard answer to the second question above is given in sects. 37.1-37.5, pp. 273-279 (sects. 28.1-28.5 in 1st ed., pp. 252-259);

On an Alleged Refutation of Hilbert's Program Using Gödel's First Incompleteness Theorem
Journal of Philosophical Logic
Author: Detlefsen, Michael; **Type:** Article; **Pages:** 343 - 377; **Publication Date:** 1990; **Volume:** 19; **Issue:** 4; **Tags:** Available online, Recommended reading; **Public note:** A dissenting voice. Reprinted in his Proof, Logic and Formalization (London: Routledge, 1991), ch. 8.;

The Concept of Infinity in Mathematics
Author: Gentzen, Gerhard; **Type:** Book Chapter; **Book Title:** The Collected Papers of Gerhard Gentzen; **Book Author:** Gentzen, Gerhard; **Chapter Number:** 6; **Pages:** 223 - 233; **Editor:** M.E. Szabo, M.E.; **Publication Date:** 1969; **Publisher:** North-Holland; **Place of publication:** Amsterdam; **Tags:** Available online, Further reading;

Hilbert's Program Revisited
Synthese
Author: Raatikainen, Panu; **Type:** Article; **Pages:** 157 - 177; **Publication Date:** 2003; **Volume:** 137; **Issue:** 1/2; **Tags:** Available online, Further reading;

Partial Realizations of Hilbert's Program
The Journal of Symbolic Logic
Author: Simpson, Stephen G.; **Type:** Article; **Pages:** 349 - 363; **Publication Date:** 1988; **Volume:** 53; **Issue:** 2; **Tags:** Available online, Further reading; **Public note:** For enthusiasts who want to know something of the afterlife of Hilbert's Programme;

Hilbert's Program
Author: Zach, Richard; **Additional Person Name:** edited by Edward N. Zalta; **Type:** Website; **Publisher:** The Stanford Encyclopedia of Philosophy (Summer 2019 Edition); **Tags:** Available online, Further reading;

Hilbert's Program Then and Now
Type: Book Chapter; **Chapter Author:** Zach, Richard; **Book Title:** Philosophy of Logic; **Pages:** 411 - 447; **Editor:** Jacquette, Dale; **Publication Date:** 20056; **Publisher:** Elsevier; **Place of publication:** Amsterdam; **Tags:** Available online, Further reading;

Recursive Functions and Computability

Expositions
What is Turing computable function? What is recursive function? Why are they the same class of functions?

An Introduction to Gödel's Theorems
Author: Smith, Peter; **Type:** Book; **Edition:** 2nd ed.; **Publication Date:** 2013; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Recommended reading; **Public note:** Chapters 38, 41 and 42 in 2nd ed. (chapters 29, 31 and 32 in 1st ed.);

Computability and Logic
Author: Boolos, George S., et al.; **Type:** Book; **Edition:** 5th ed.; **Publication Date:** 2007; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Recommended reading; **Public note:** Alternative reading. Chapters 3-8. Though many think the treatment of the same chapters of the 3rd ed. - when the authors were just Boolos and Jeffrey - is nicer.;

Computability: An Introduction to Recursive Function Theory
Author: Cutland, Nigel; **Type:** Book; **Publication Date:** 1980; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Recommended reading; **Public note:** Another alternative reading. A classic book that will appeal to mathematicians;

Computability, Unsolvability, Undecidability
Author: Hamilton, A. G.; **Type:** Book Chapter; **Book Title:** Logic for Mathematicians; **Book Author:** Hamilton, A. G.; **Chapter Number:** 7; **Pages:** 156 - 198; **Publication Date:** 1978; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Recommended reading, Scan available;

Theory of Recursive Functions and Effective Computability
Author: Rogers, Hartley; **Type:** Book; **Publication Date:** 1987; **Publisher:** MIT Press; **Place of publication:** Cambridge; **Tags:** Available online, Recommended reading, Scan available; **Public note:** An old classic from 1967 which is well worth reading the first chapter of, especially sects. 1.1-1.7. Scan is of the whole of Chapter 1 Recursive Functions;

Philosophical issue arising: What is the status of Church's thesis?
It is a mathematical theorem that a function is Turing computable if and only if it is recursive (if and only if it register computable, if and only if it is Herbrand-Gödel computable etc.). Different attempts to regiment the intuitive notion of a computable function all converge. Church's Thesis (a.k.a. the Church-Turing Thesis) claims that indeed the intuitively computable functions are just the Turing computable/recursive functions.

Turing Machines and Recursiveness
Author: Smith, Peter; **Type:** Book Chapter; **Book Title:** An Introduction to Gödel's Theorems; **Book Author:** Smith, Peter; **Pages:** 321 - 327; **Edition:** 2nd ed.; **Publication Date:** 2013; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Recommended reading; **Public note:** Read this first for some initial clarifications. Chapter 42 in 2nd ed. (chapter 32 in 1st ed.);

Understanding Church's Thesis
Journal of Philosophical Logic
Author: Shapiro, Stewart; **Type:** Article; **Pages:** 353 - 365; **Publication Date:** 1981; **Volume:** 10; **Issue:** 3; **Tags:** Available online, Recommended reading; **Public note:** Read after Peter Smith;

Proving Church's Thesis
Philosophia Mathematica
Author: Black, Robert; **Type:** Article; **Pages:** 244 - 258; **Publication Date:** 2000; **Volume:** 8; **Issue:** 3; **Tags:** Available online, Recommended reading; **Public note:** Read after Peter Smith;

Computability, Proof and Open-Texture
Type: Book Chapter; **Chapter Author:** Shapiro, Stewart; **Book Title:** Church's Thesis After 70 Years; **Pages:** 420 - 455; **Editor:** A. Olszewski, et al.; **Publication Date:** 2006; **Publisher:** Ontos; **Place of publication:** Frankfurt; **Tags:** Available online, Recommended reading; **Public note:** Read after Peter Smith;

Halting and Incompleteness
Author: Smith, Peter; **Type:** Book Chapter; **Book Title:** An Introduction to Gödel's Theorems; **Book Author:** Smith, Peter; **Chapter Number:** 43; **Pages:** 328 - 337; **Edition:** 2nd ed.; **Publication Date:** 2013; **Publisher:** Cambridge University Press; **Place of publication:** Cambridge; **Tags:** Available online, Recommended reading; **Public note:** Takes an opinionated minority line. In first ed. (2007) - chapter 33 has different title (Halting Problems). Read after chapter on Turing.;

The Church-Turing Thesis
Author: Copeland, B. Jack; **Additional Person Name:** edited by Zalta, Edward N.; **Type:** Website; **Publisher:** The Stanford Encyclopedia of Philosophy (Spring 2019 Edition); **Tags:** Available online, Further reading;

Second Thoughts about Church's Thesis and Mathematical Proofs
The Journal of Philosophy
Author: Mendelson, Elliott; **Type:** Article; **Pages:** 225 - 233; **Publication Date:** 1990; **Volume:** 87; **Issue:** 5; **Tags:** Available online, Further reading;

Review of Church's Thesis after 70 Years
Author: Smith, Peter; **Type:** Article; **Publication Date:** 2007; **Tags:** Available online, Further reading;

Non-Turing Computers and Non-Turing Computability
Proceedings of the Biennial Meetings of the Philosophy of Science Association
Author: Hogarth, Mark; **Type:** Article; **Pages:** 126 - 138; **Publication Date:** 1994; **Issue:** 1; **Tags:** Available online, Further reading; **Public note:** Part of an interesting local sub-debate;

Deciding Arithmetic Using SAD Computers
The British Journal for the Philosophy of Science
Author: Hogarth, M; **Type:** Article; **Pages:** 681 - 691; **Publication Date:** 2004; **Volume:** 55; **Issue:** 4; **Tags:** Available online, Further reading; **Public note:** Part of an interesting local sub-debate;

SAD Computers and Two Versions of the Church–Turing Thesis
The British Journal for the Philosophy of Science
Author: Button, Tim; **Type:** Article; **Pages:** 765 - 792; **Publication Date:** 2009; **Volume:** 60; **Issue:** 4; **Tags:** Available online, Further reading; **Public note:** Part of an interesting local sub-debate [Criticizes Hogarth];

Suggestions Welcome
We welcome your suggestions for further readings that will improve and diversify our reading lists, to reflect the best recent research, and important work by members of under-represented groups. Please email your suggestions to philosophy@lib.cam.ac.uk including the relevant part and paper number. For information on how we handle your personal data when you submit a suggestion please see: https://www.information-compliance.admin.cam.ac.uk/data-protection/general-data.